NONLINEAR AND NONCLASSICAL PROPERTIES OF DEFORMED QUANTUM STATES

A thesis submitted

in partial fulfillment for the award of the degree of

Doctor of Philosophy

by

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April 2023

Abstract

We begin our work by studying the most general class of oscillators, called 'f-deformed oscillators' or 'f-oscillators'. We define the quadrature operator for the f-deformed algebra and hence obtain the deformed quadrature operator eigenstates. We derived a new set of polynomials and derived the deformed oscillator wavefunctions in terms of them. The position probability distributions for three different types of deformations are plotted, and each is compared with the corresponding non-deformed counterpart. The newly obtained quadrature operator eigenstates will be helpful for those who are working in the field of quantum state reconstruction and quantum information processing of deformed states.

Later, we focus mainly on one of the special cases of f-deformation, i.e., the math-type q-deformation. We inquire into the nonclassical properties of the math-type q-deformed states. Here we report the study of squeezing in q-deformed squeezed vacuum states, their superposition, and the superposition of q-deformed squeezed coherent states of a math-type q-deformed oscillator. Quadrature squeezing, higher-order squeezing, and number squeezing are studied. The analysis reveals that the states exhibit squeezing only for a specific range of the deformation parameter q and the squeezing parameter r. We find that the quadrature squeezing coefficient is independent of q, in the q-deformed squeezed vacuum states. The squeezing vanishes when we go for their superposition. We also studied another nonclassical property, the Husimi Q function, for the above mentioned states. Husimi Q function reveals that these states are highly nonclassical irrespective of squeezing. The nonclassicality present in the deformed states is found to be dependent on q, α and r.

We then extend our study of the math-type q-deformed oscillator into its dynamical behavior by analyzing its expectation values. A primary analysis of the system's dynamics hints at the possibility of chaos in it. Although the search for chaos in quantum systems has been an area of prominent research over the last few decades, the detailed analysis of many inherently chaotic quantum systems based on expectation values of dynamical variables has not been reported in the literature. The system is found to be periodic, quasi-periodic, or chaotic depending on the values of the deformation parameter q and the deformed coherent amplitude α, thus enabling us to explicitly classify the chaotic nature of the system based

on these parameters. A further detailed study using recurrence plots, power spectra, first-return-time distributions, and Lyapunov exponents unambiguously confirmed the chaotic behavior of the system, existing over a specific range of q and α values.

We outstretch the study of the nonclassicality of a math-type q-deformed system to quantum entanglement, one of the most discussed and relevant areas of quantum physics. We deal with the propagation of a single-mode math-type q-deformed field through a nonlinear medium, where the atoms of the medium interact with the deformed field. We measure the entanglement in terms of von Neumann entropy, and its temporal evolution shows that the states are entangled for all the possible deformation values. We have considered the system in two different initial states, whose dynamics exhibit near revivals and fractional revivals. But the revivals die out even for a slight deformation increase. Thus here we provide the analysis of a general deformed system, with an additional degree of freedom, i.e., the deformation parameter q to control its nonlinear and nonclassical properties.