Halo Orbit Design Around Lagrangian Points Using Gradient and Non Gradient Based Optimization Techniques

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Introduction

- The design of space missions involve the dynamics of multi-bodies, solution of which is complex.
- With some restrictive assumptions, insights necessary for the preliminary trajectory design can be obtained.
- The Circular Restricted Three Body Problem (CRTBP) is one such framework and provides a good initial approximation to start the real mission design involving full force ephemeris models. In CRTBP, the third body is assumed to move in the plane formed by two bodies which revolve around their center of mass in circular orbits.
- Under this framework, there are five points known as Lagrangian points where the gravitational and centrifugal accelerations of the third body balance each other. Families of periodic orbits are known to exist around Lagrangian points of which halo orbits form an important class.
- The mission design to a halo orbit around Lagrangian points from Earth involves, as a first step, the design of the halo orbit of a specified size i.e. the out of plane amplitude, *Az*.
- This paper deals with the halo orbit design using Differential Correction (DC - Gradient based) and Differential Evolution (DE - Non Gradient based) optimization techniques. These two approaches are compared and contrasted on the basis of computational effort, ability to meet the required constraints and scope for further improvements. To get better performance from the DE based process, two modified variants of DE are employed.

Governing Dynamics

- A coordinate frame whose origin is fixed at the center of mass of the primaries and which rotates with the rotation of primaries is introduced. For normalization, it is assumed that the distance between the primaries is unity and the sum of masses of the primaries is also unity.

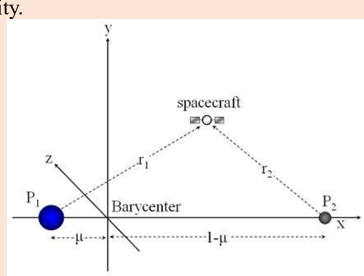

Figure 1: Barycentric rotating coordinate frame in CRTBP

- The motion of the third body in this coordinate system are governed by the following equations [1].

$$\ddot{x} = x + 2\dot{y} - \frac{1-\mu}{r_1^3}(x+\mu) - \frac{\mu}{r_2^3}\left(x-(1-\mu)\right) \quad (1)$$

$$\ddot{y} = y - 2\dot{x} - \frac{1-\mu}{r_1^3}y - \frac{\mu}{r_2^3}y \quad (2)$$

$$\ddot{z} = -\frac{1-\mu}{r_1^3}z - \frac{\mu}{r_2^3}z \quad (3)$$

- The halo orbit crosses the X-Z plane orthogonally. If the initial state is $[x_0, 0, z_0, 0, \dot{y}_0, 0]$ the state at the half period is $[x_{T/2}, 0, z_{T/2}, 0, \dot{y}_{T/2}, 0]$
- For the halo orbit design, suitable initial conditions that lead to orthogonal crossing at half period must be obtained.

Halo Orbit Design Using Differential Correction (DC)

- The solution to the six linearized state variational equations is expressed in terms of State Transition Matrix (STM). The 6X6 STM represents the sensitivity of the initial state to the final state and consists of 36 differential equations.
- Differential Correction scheme uses the STM to determine the changes to the initial conditions which nullify the deviations in the *x* and *z* velocity components at the X-Z plane crossing.
- The objective function '*OBJ*' is set as:

$$OBJ = |\dot{x}_{T/2}| + |\dot{z}_{T/2}|$$

- The STM is initiated to be an identity matrix and a good guess is made on the initial state based on third order theory [2]. The system of 42 differential equations are numerically integrated, till X-Z plane crossing and deviations are noted.
- The process is repeated by updating the initial state till a pre-defined tolerance.

Table 1: Achieved *Az* amplitudes using DC process

Az desired (km)	*Az* achieved (km)	Computational time (s)	No of Iterations
120000	119358.42	0.004	5
400000	396995.62	0.004	5
750000	736125.19	0.006	5
900000	874195.26	0.006	5

- The DC procedure doesn't give the required out of plane amplitude.
- In order to overcome this problem, an alternate scheme based on Differential Evolution has been employed.

Halo Orbit Design Using Differential Evolution (DE)

- DE is a heuristic direct search method that mimics the evolution of living species [3].
- It requires only bounds for the unknown parameters.
- In halo orbit design, the choice of bounds for the unknown parameters are made on the basis of dynamics involved in the problem.
- The *x* and *z* velocity components at X-Z plane crossing need to be zeros. In order to accomplish this and to meet the requirement of desired *Az* amplitude, the following objective function '*OBJ*' is set as: [4].

$$OBJ = |\dot{x}_{T/2}| + |\dot{z}_{T/2}| + |Az_{achieved} - Az_{desired}|$$

- The initial population consists of values for the three unknowns (randomly chosen from their respective bounds) and the corresponding objective function. For objective function, numerical integration of the equations of motion (Eq. 1-3) is done using Runge-Kutta 4th order integrator till the *X-Z* plane crossing.
- Each row of the current population is updated by generating a trial element based on three operations: mutation, cross over and selection.
- The trial element will replace the current element in the next population if the objective function value is lesser.
- The above mentioned steps are repeated till the value of objective function is less than a pre-fixed tolerance. For this problem, the tolerance value is 1.0E-15.

Table 2: DE performance for different population sizes.

Population size (N)	No of iterations	Computational time (s)
30	1920	165.812
40	261	69.380
50	279	89.044
60	262	106.836
70	264	126.444

- DE performance has been assessed for parameters such as mutation factor (F) and cross over ratio (CR) for *Az* = 120000 km. The selected parameters are N=40, F=0.5 and CR=0.8.
- The converged solution for all the cases is : *x0*=0.98883722109728, *z0*=0.000815222291719, *y0*=0.008940530087256, *Az* achieved = 109999.99999999678 km
- The desired *Az* is achieved (110000 km).
- Globality and robustness of the solution is established by varying seeds (that represent random sequences) and bounds for initial conditions. DE converged to the same solution in all scenarios.
- To reduce the computational time and get better performance two modified variants of DE are employed.

Halo Orbit Design Using Modified Differential Evolution

- The mutation process in basic DE (Scheme 1) is modified as follows:

Scheme 2: $v_{G+1} = x_{best,G} + F.(x_{r1,G} + x_{r2,G} - x_{r3,G} - x_{r4,G})$

Scheme 3: $v_{G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G}) + F.(x_{best,G} - x_{r3,G})$

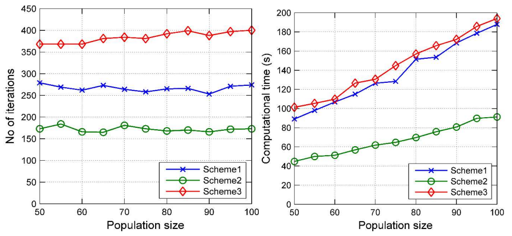

Figure 2: Performance of DE schemes in halo orbit design (*Az* = 110000 km).

- The Scheme 2 performs better than Scheme 1 and Scheme 3 in terms of computational time and effort, probably because it generates the trial vector around the vector of lowest cost from the population.

Conclusions

- Halo orbit design around Sun-Earth Lagrangian points is carried out using Differential Correction and Differential Evolution techniques.
- The DC procedure doesn't give the required *Az* amplitude.
- The DE based algorithm provides precise design in a single level process.
- Two improved variants of DE in terms of mutation strategies are employed. It is observed that the Scheme 2 performs better than all others in computational time and effort.

References

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